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PRINCIPLE OF LOCAL INFLUENCE IN THE METHOD OF STEP BY STEP MODELING

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UDC 536.2

The possibility of replacing spatially inhomogeneous effect on body boundaries by average effects is investigated. Error estimates are obtained for a temperature field computation.

The crux of the step-by-step modeling of the thermal mode of complex systems is the subsequent application of a number of mathematical models possessing a different degree of detailing in the description of temperature fields [1]. In the initial stages the thermal regime of the whole system is analyzed with minimally allowable detailing and the average values of the temperature and heat fluxes are determined. The following approach is used in computing the temperature field of a certain isolated domain of the system: the boundary conditions for the domain under consideration are compiled on the basis of values of the temperature and heat fluxes obtained in the preceding stages of the computation.

For a strict assignment of the boundary conditions it is necessary to know the spatial temperature distribution $T(x)$ or the heat flux density $q(x)$ on the boundary of the isolated domain, or in the case of boundary conditions of the third kind, the temperature distribution of the conditions medium $T_m(x)$. However, the models of the previous stages that possess a smaller degree of detailing permit the computation of just the temperature values $\langle T_k \rangle$ and $\langle q_k \rangle$ averaged over certain surfaces (or coordinates)

$$\langle T_k \rangle = \frac{1}{S_k} \int_{S_k} T(\mathbf{x})\, dS_k, \quad \langle q_k \rangle = \frac{1}{S_k} \int_{S_k} q(\mathbf{x})\, dS_k. \tag{1}$$

Hence, in the step-by-step modeling strict formulation of the boundary-value problem for an isolated domain of the system is replaced by an approximate formulation yielding the mean temperature on the sections Γ_k

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$$T|_{\Gamma_k} = \langle T_k \rangle . \quad (2)$$

or the mean heat flux density

$$-\lambda \frac{\partial T}{\partial n}\Big|_{\Gamma_k} = \langle q_k \rangle . \quad (3)$$

or the mean temperature of the environment

$$\left[\lambda \frac{\partial T}{\partial n} + \alpha_k (T - \langle T_{mk} \rangle)\right]_{\Gamma_k} = 0. \quad (4)$$

Let us note that the method described for assigning the boundary conditions was used in many papers on the computation of temperature fields in complex technical objects [2-4]. However, we did not succeed in detecting investigations in the literature on estimates of the error in the computation induced by this approximation. The legitimacy of replacing the spatial temperature and density of the heat flux by their values averaged over the section of the boundary was based qualitatively on the principle of local influence [5], according to which the temperature distribution at a sufficient distance from the boundary should be insensitive to such perturbations on the boundary. In this paper an attempt is made to obtain quantitative error estimates of the temperature field computation caused by averaging the boundary conditions. The dependence of this error on factors characterizing the geometric parameters of the domain and the heat-exchange conditions on the boundaries is investigated. The analysis is for linear stationary problems with heat conductivity λ and heat elimination coefficients α_k independent of the coordinates on the sections Γ_k of the boundary.

We first consider the general formulation of the problem, and then we turn to an investigation of the error for the model problems. Compile the solution of the stationary heat-conduction equation

$$\lambda \nabla^2 T + q_V = 0 \quad (5)$$

in a certain domain D for two methods of giving the boundary conditions on the sections Γ_k ($k = 1, \ldots, N$) of the boundary:

the temperatures of the environment depend on the coordinates

$$\left[\lambda \frac{\partial T}{\partial n} + \alpha_k (T - T_{mk}(\mathbf{x}))\right]_{\Gamma_k} = 0, \quad (6)$$

temperatures of the media averaged over the corresponding boundary sections are in the boundary conditions, i.e., conditions (4) are given.

We investigate the error for boundary conditions of the third kind since the results obtained can be extended to boundary conditions of the first kind by letting $\alpha \to \infty$ and to conditions of the second kind by passing to the limit as $\alpha \to 0$, $T_{mk} \to \infty$ and $\alpha T_{mk} = q_k = \text{const}$.

The solution of problems (5), (6) and (5), (4) can be represented in the form

$$T(\mathbf{x}) = T_n(\mathbf{x}) + \sum_{k=1}^{N} T_{0k}(\mathbf{x}), \quad (7)$$

where $T_n(x)$ is the solution of the nonhomogeneous equation (5) for the homogeneous boundary conditions ($T_{mk} = 0$) and $T_{0k}(x)$ is the solution of the homogeneous equation ($q_V = 0$) for a nonhomogeneous boundary condition on the section Γ_k and homogeneous boundary conditions on the remaining sections of the boundary Γ_j $(j=1, \ldots, N,\ k \neq j)$.

Hence, to estimate the error caused by averaging the temperature of the medium, it is sufficient to consider the solution of the homogeneous equation

$$\nabla^2 T(\mathbf{x}) = 0, \ \mathbf{x} \in D, \quad (8)$$

with homogeneous boundary conditions on the sections Γ_j of the boundary of the domain D under consideration

$$\left[\lambda \frac{\partial T}{\partial n} + \alpha_j T\right]_{\Gamma_j} = 0, \ j = 1, \ldots, N, \ j \neq k, \quad (9)$$

and nonhomogeneous conditions (4) or (6) on one section of the boundary Γ_k. The error of a temperature field computation for simultaneous perturbations of the boundary conditions on all sections Γ_j can be found by using the superposition principle in conformity with (7).

We denote the solution of the problem (8), (9), (6) by $T_1(x)$ and of the problem (8), (9), (4) with averaged temperature of the medium by $T_2(x)$. The error $u(x) = T_1(x) - T_2(x)$ satisfies (8), homogeneous boundary conditions (9), and the following condition on the boundary

$$\left[\lambda \frac{\partial u}{\partial n} + \alpha_k u\right]_{\Gamma_k} = \alpha_k \Delta T_{mk}(\mathbf{x}), \tag{10}$$

where $\Delta T_{mk}(x) = T_{mk}(x) - \langle T_{mk}\rangle$ is the deviation of the temperature of the medium from the mean value, later called the perturbation of the temperature of the medium.

We turn to a dimensionless formulation of the problem by selecting the characteristic dimension l_k of the domain for averaging the boundary conditions Γ_k as the length scale and the maximal value of the perturbation ΔT_{mk} as the error scale. We introduce the following dimensionless quantities: the relative error

$$\theta(\mathbf{x}) = u(\mathbf{x})/\max_{\mathbf{x}\in\Gamma_k} |\Delta T_{mk}(\mathbf{x})|, \tag{11}$$

the relative perturbation of the temperature of the medium

$$\theta_{mk}(\mathbf{x}) = \Delta T_{mk}(\mathbf{x})/\max_{\mathbf{x}\in\Gamma_k} |\Delta T_{mk}(\mathbf{x})|, \tag{12}$$

the relative coordinate, $\mathbf{x}' = \mathbf{x}/l_k$, the Biot criterion $Bi_j = \alpha_j l_k/\lambda$, $j = 1, \ldots, N$.

Then we write the problem of determining the relative error $\theta(\mathbf{x}')$ in the form

$$\nabla^2\theta = 0, \ \mathbf{x} \in D, \tag{13}$$

$$\left[\frac{\partial\theta}{\partial n'} + Bi_k\,\theta\right]_{\Gamma_k} = Bi_k \theta_{mk}(\mathbf{x}'), \tag{14}$$

$$\left[\frac{\partial\theta}{\partial n'} + Bi_j\,\theta\right]_{\Gamma_j} = 0, \ j = 1, \ldots, N, \ j \neq k. \tag{15}$$

The relative error $\theta(x')$ caused by averaging the temperature of the medium depends on the form of the perturbation $\theta_{mk}(x')$, on the criteria Bi_j ($j = 1, \ldots, N$) and on the geometric factors, the number and form of which is determined by the dimension and configuration of the domain D under consideration.

Let us consider the passage to a dimensionless formulation of the problem in the case when boundary conditions of the second kind are given on the section Γ_k

$$\lambda \left.\frac{\partial T}{\partial n}\right|_{\Gamma_k} = q_k, \text{where } q_k = q_k(\mathbf{x}) \quad \text{or} \quad q_k = \langle q_k \rangle .$$

The solution $T_1(\mathbf{x})$ obtained for $q_k(\mathbf{x})$ is compared with the solution $T_2(\mathbf{x})$ for the averaged flux $\langle q_k\rangle$. We determine the relative error θ' as follows:

$$\theta'(x') = \frac{\lambda\,[T_1(\mathbf{x}) - T_2(\mathbf{x})]}{l_k \max |q_k(\mathbf{x}) - \langle q_k \rangle|}, \tag{16}$$

and the boundary condition on Γ_k in the problem for the relative error takes the form

$$\left.\frac{\partial\theta'}{\partial n'}\right|_{\Gamma_k} = \Delta q_k'(\mathbf{x}'), \ \Delta q_k' = \frac{q_k(\mathbf{x}) - \langle q_k \rangle}{\max |q_k(\mathbf{x}) - \langle q_k \rangle|}. \tag{17}$$

It is easy to show that the relative error θ' can be found from the solution of the problem (13)-(15) by passing to the limit

$$\theta'(\mathbf{x}') = \lim_{Bi_k \to 0} [\theta(\mathbf{x}')/Bi_k] \tag{18}$$

and setting $\Delta q_k'(\mathbf{x}') = \theta_{mk}(\mathbf{x}')$.

The error estimate for the boundary conditions of the first kind (2) can be obtained directly from the solution of the problem (13)-(15) for sufficiently large Bi_k assuring $\theta|_{\Gamma_k} \approx \theta_{mk}$.

The solution of problem (13)-(15) is analyzed below for different domains and recommendations are given on the error estimate in averaging the temperatures in the boundary conditions.

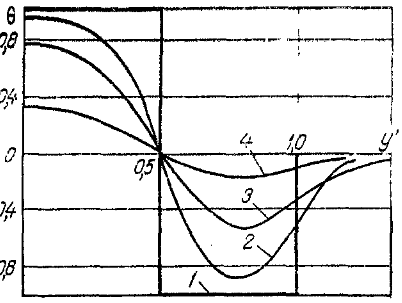

Fig. 1

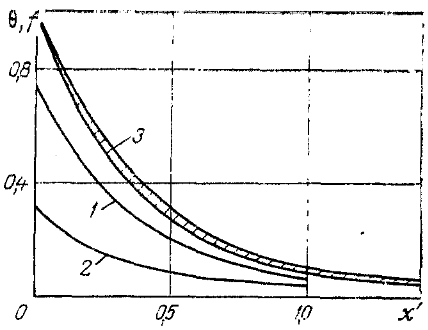

Fig. 2

Fig. 1. Distribution of the relative error θ(0, y') on the boundary x' = 0 of a semiinfinite body: 1) Bi → ∞; 2) Bi = 20; 3) 5; 4) 1.

Fig. 2. Change in the maximum relative error with distance from the boundary: 1) Bi = 5; 2) Bi = 1; 3) domain of values f(x') = θ(x', 0)/θ(0, 0).

We start with the consideration of the two-dimensional problem for a semiinifite body $(0 \leqslant x' < \infty, -\infty < y' < \infty)$. On the surface x' = 0 a boundary condition of the third kind is given in which the temperature of the environment has the relative perturbation $\theta_m(y')$ on a certain bounded section. The relative error θ(x', y') is the solution of the problem

$$\frac{\partial^2\theta}{\partial x'^2} + \frac{\partial^2\theta}{\partial y'^2} = 0, \tag{19}$$

$$\left[-\frac{\partial\theta}{\partial x'} + \mathrm{Bi}\,\theta\right]_{x'=0} = \mathrm{Bi}\,\theta_m(y'), \tag{20}$$

$$\left.\frac{\partial\theta}{\partial x'}\right|_{x'\to\infty} = 0, \quad \frac{\partial\theta}{\partial y'}_{|y'|\to\infty} = 0. \tag{21}$$

Comparing the solutions of problems (19)-(21) for different perturbations $\theta_m(y')$ showed that the error reaches the greatest values for the assignment of the following kind of relative temperature perturbation of the medium:

$$\theta_m(y') = \begin{cases} 1, & 0 \leqslant |y'| < 0.5, \\ -1, & 0,5 < y' \leqslant 1, \\ 0, & |y'| > 1,\ |y'| = 0.5. \end{cases} \tag{22}$$

An exact analytic solution of the problem (19)-(22), obtained by using the Fourier cosine transform [6], has the form

$$\theta = \frac{4\,\mathrm{Bi}}{\pi}\int_0^\infty \frac{(1-\cos\omega/2)\sin\omega/2}{\omega(\omega+\mathrm{Bi})}\exp(-\omega x')\cos\omega y'\,d\omega. \tag{23}$$

Analysis of solution (23) follows two ends: firstly, to investigate how the temperature perturbation of the medium is "mapped" on the body surface for a different intensity of the heat transfer with the medium, i.e., to find the dependence of the relative error θ(x' = 0) on the criterion Bi; secondly, to estimate the "damping" of the relative error with distance from the boundary into the bulk of the body.

The distribution of the relative error θ(0, y') on the body boundary is shown in Fig. 1 for different Bi. The error for the selected form of perturbation is an even function of y' and its maximal value is achieved at y' = 0. The change in the maximum value of the relative error $\theta_{max}(x')$ with distance from the boundary is displayed in Fig. 2. The magnitude of the error depends on the criterion Bi, hence, to characterize the diminution in error with distance from the boundary, we introduce the quantity f(x') = θ(x', 0)/θ(0, 0) which is the ratio between the maximal error in the plane x' and its value on the boundary x' = 0. Dependences f(x') constructed for different Bi are sufficiently close together. The domain in which the f(x') lie for Bi = 0.1-20 is displayed in Fig. 2.

The dependence of the maximal relative error θ(0, 0) at the boundary on Bi is represented in Fig. 3 (curve 1). For Bi < 0.2 the error becomes proportional to Bi, hence, in the domain of small Bi not represented on the graphs, the quantity θ can be calculated easily.

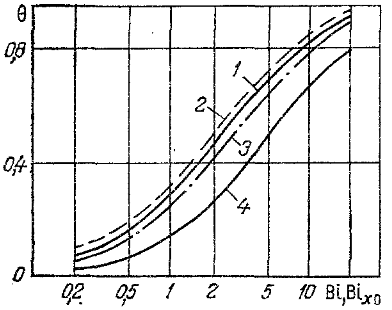

Fig. 3

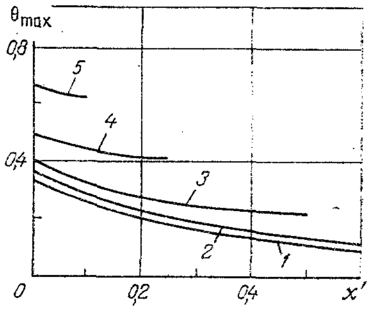

Fig. 4

Fig. 3. Dependence of the maximal relative error on the heat-transfer conditions on the boundary: 1) semiinfinite body; 2, 3, 4) rectangular domain as $h \to \infty$ (2) $Bi_{y0} = 0$; $Bi_{y1} \to \infty$; 3) $Bi_{y0} = Bi_{y1} = 0$; 4) $Bi_{y0} \to \infty$, $Bi_{y1} \to \infty$).

Fig. 4. Distribution of the maximal relative error $\theta_{max}(x')$ in a rectangular domain: 1) $h \to \infty$; 2) $h = 1$; 3) 0.5; 4) 0.25; 5) 0.1.

To estimate the error induced by averaging the temperature of the medium on a section of surface of a simiinfinite body, a formula is proposed that approximates the results obtained

$$\theta_{max}(x') = \frac{Bi}{2{,}2 + Bi} \exp(-2{,}3x'), \quad 0.2 \leqslant Bi \leqslant 20, \quad 0 \leqslant x' \leqslant 3. \tag{24}$$

The error θ' caused by averaging the heat flux density (see (16)) can be estimated from the formula

$$\theta_{max}(x') = 0{,}45 \exp(-2{,}1x'), \quad 0 \leqslant x' \leqslant 3. \tag{25}$$

If the dimensions of the boundary conditions perturbation domain are commensurate with the body dimensions, then in estimate the error the model of a semiinfinite body can turn out to be inapplicable. The error in averaging the boundary conditions depends on the heat-transfer conditions on the other boundaries and on the body configuration in this case. We find the singularities of propagating the boundary condition perturbations in a bounded body from an examination of the following problem. A two-dimensional rectangular domain is given with the dimensions l_x, l_y on whose boundaries heat transfer occurs with the environment. The temperature of the medium is averaged on the boundary $x = 0$. The relative error $\theta(x', y')$ is found from the solution of (19) with the boundary conditions

$$\left[-\frac{\partial \theta}{\partial x'} + Bi_{x0}\theta\right]_{x'=0} = Bi_{x0}\,\theta_m(y'), \tag{26}$$

$$\left[\frac{\partial \theta}{\partial x'} + Bi_{x1}\theta\right]_{x'=h} = 0, \quad \left[\mp\frac{\partial \theta}{\partial y'} + Bi_{y0,y1}\,\theta\right]_{y'=0,1} = 0, \tag{27}$$

where $Bi_k = \alpha_k l_y/\lambda$, $k = x0, x1, y0, y1$, $x' = x/l_y$, $y' = y/l_y$, $h = l_x/l_y$.

The perturbations $\theta_m(y')$ is given in form (22). An exact analytic solution of the problem (19), (26), (27) is obtained by using the Fourier integral transform in the variable y' [6].

Dependences of the maximal relative error on the boundary $x' = 0$ on the criterion Bi_{x0} as $h \to \infty$ are represented in Fig. 3 for different limit cases of the heat-exchange conditions on the boundaries $y' = 0$ and $y' = 1$. The error reaches the highest values under the conditions $Bi_{y0} = 0$, $Bi_{y1} \to \infty$ and the least for $Bi_{y0} \to \infty$, $Bi_{y1} \to \infty$. For an approximate estimate of the maximal error, the results obtained for a semiinfinite body (curve 1) can be used.

The change in the maximum relative error with distance from the boundary $x' = 0$ is shown in Fig. 4 for different relationships between the body dimensions $h = l_x/l_y$. For $h > 1$ the error distribution depends negligibly on the heat-transfer conditions on the boundary $x' = h$ and on the quantity h, consequently, results for the case $h \to \infty$ can be used. For small h the influence of the boundary $x' = h$ becomes substantial, where the highest errors occur in the case of the adiabatic condition $Bi_{x1} = 0$.

The results obtained yield quantitative estimates of the errors associated with replacement of the spatial temperature distributions of the medium and of the heat-flux density of their values averaged over the boundary section, which permits a well-founded approach to the selection of algorithms of the step-by-step modeling of the thermal regime of a system of bodies.

NOTATION

T, T_m, temperature of the body and the conditional medium; q, heat-flux density; λ, heat conductivity; α, heat-transfer coefficient; q_V, specific power of the heat sources; x, radius-vector; θ, relative error in computing the temperature under average boundary conditions; l_k, governing dimension of a section of the boundary Γ_k; x', y', relative coordinates; Bi, Biot criterion.

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APPROXIMATE SOLUTION OF THE STEFAN PROBLEM ON A SEGMENT

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The behavior of the temperature and the boundary near the stationary state are studied in the single-phase Stefan problem for certain types of thermal flux variations.

The perturbation of the stationary solution of the Stefan problem on a segment for small changes in the flux acting on the boundary was examined in [1, 2], where general expressions are obtained for the boundary and temperature for $0 \leqslant t < \infty$*. A detailed investigation of this approximate solution is quite important in applications but it is difficult for arbitrary flux perturbations. The most characteristic cases (step and sinusoidal thermal flux variation) are analyzed in detail in this paper, hence, asymptotic formulas are obtained for the solution for "small" and "large" times. The general solution of the problem under consideration is also simplified for slow and smooth flux changes.

The problem is formulated as follows. Find the classical solution of a system of equations with additional conditions

*The method used to obtain this solution was also applied in the two-phase problem [3].

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